# MATH 4032, SPRING 2024: PROJECT 3

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For this project, you will choose a topic related to extremal combinatorics and write an expository report on it. The purpose of the report should be akin to introductory lecture notes, where the target audience is another student in our class: provide motivation for studying the topics, any necessary background or definitions, guiding examples, and a main theorem or theorems and proofs.

## 1. Topics

1.1. **List of Potential Topics.** See bibliography for hyperlinks to suggested starting references, all of which are available online.

Ramsey Theory:

- Schur's theorem (Jukna [5, Theorem 4.14])
- arithmetic progressions and van der Waerden numbers (Jukna [5, Section 25.1])
- combinatorial lines and Hales-Jewett Theorem (Jukna [5, Chapter 26])
- infinite graphs and canonical Ramsey numbers (lecture notes [1, Section 1])
- partition regularity and Rado's theorem (lecture notes [1, Section 3])
- topological dynamics in Ramsey theory, Hindman's theorem (lecture notes [1, Section 4])
- (challenge) stepping-up method for hypergraph Ramsey numbers (lecture notes [2])

Extremal Graph Theory:

- dependent random choice (survey paper [3])
- (challenge) regularity lemma (Zhao [8, Section 2.1])

Extremal Set Theory:

- the sunflower lemma (Jukna [5, Chapter 6])
- VC dimension and Perles-Sauer-Shelah lemma (Jukna [5, Chapter 10])
- (challenge) union-closed sets conjecture (blog post [6] and paper [4])

See also [7] for an older set of Ramsey theory lecture notes that covers much of the same material as [1]. Other textbook references include *Ramsey Theory* by Graham-Rothschild-Spencer and Chapter 7 of *Graph Theory* by Diestel but are somewhat more challenging to read.

1.2. **A Note on Topic Choice.** The topics above are suggested because I think they are the most accessible based on what we have covered in class and also lend themselves to a lot of specific examples. You are welcome to propose something else related to extremal combinatorics, but please let me know so we can discuss if it would be suitable for this project.

## 2. Requirements of Report

2.1. **Formatting.**

- Length: 2–4 pages
- Format: typed, single-spaced in 11–12pt font (this is the LaTeX default) with 1-inch margins
- I *strongly encourage you* to write this report in LaTeX but it is not required.

- At the end should be a bibliography. An easy way to do this is to create a separate .bib file (in the same folder on your computer, or in the same project on Overleaf) and then use the code (for example) `\bibliographystyle{plain} \bibliography{name of .bib file}`. Here are some instructions about how to do this.

2.2. **Content.**

- An introduction section, including motivation for studying the topic
- Any new definitions or notation that we have not introduced in class
- At least one "main theorem"
- At least one mathematical proof. Note: it doesn't have to be the proof of the main theorem if, for example, the main theorem proof is too complicated to discuss in full detail.
- A conclusion
- In general, you may research anything related to the topic but please cite all sources.
- Your target audience should be other students, such as those in Math 4032, so keep that in mind when deciding what background to include.

2.3. **Deadlines.**

- Due Friday, April 5 at 11:59pm: your choice of topic
- Due Friday, April 12 at 11:59pm: an outline of your report. Examples include a set of bullet points with the key facts or theorems you will describe, a list of references, placeholders for illustrations you will insert, etc.
- Due Friday, April 19 at 11:59pm: final project.

## 3. Grading

Here are the categories according to which your project will be graded. See Canvas for a full breakdown of the rubric.

3.1. **Mathematics.** (10 pts) This category is for grading the proofs and mathematical correctness of your report. Proofs and explanations will be graded roughly according to the "Proofs" rubric used for problem sets. Namely,

- Proofs and explanations are correct, flow logically, and are easy to follow without over- or under-explaining.
- Mathematical grammar and notation are used correctly.

3.2. **Exposition and Writing.** (10 pts) This category is for grading the "essay" aspects of your report.

- The report is cohesive and has a narrative structure, as opposed to simply a list of statements or theorems.
- The report is well-organized and appropriately uses sections or headings when needed.
- The writing is clear and engaging, and targets an appropriate audience.

3.3. **Content.** (10 pts) This category is for grading how well your report balances the above two categories.

- The content of the report is clearly related to the chosen topic.
- The topics discussed in the report are sufficiently related to the class content but are not rote repetition of class content. Topics outside of the scope of class are prefaced with an appropriate amount of introduction and motivation.
- There is an appropriate balance between exposition and proofs.

3.4. **Formatting and Grammar.** (10 pts)

- The report has correct spelling, punctuation, and English grammar.
- The report follows the formatting guidelines specified in this document.
- References are appropriately cited inline and are listed in a bibliography.

3.5. **Creativity (bonus).** (3 pts)

A straightforward report that satisfies all the criteria above will earn full credit. But bonus points will be awarded if you bring something "extra" to the table or go above and beyond in some way. It can take the form of anything—an illustration, an analogy, code, a poem. But it still has to be relevant to your topic: a song about the *arctan* function, while certainly creative, would not be relevant.

## References

[1] Dexter Chua and Bhargav Narayanan. Part III Ramsey Theory Lecture Notes, 2000. URL: `https://dec41.user.srcf.net/notes/III_L/ramsey_theory_trim.pdf`.

[2] David Conlon. Ramsey theory notes, Lecture 4. URL: `https://www.its.caltech.edu/~dconlon/RamseyLecture4.pdf`.

[3] Jacob Fox and Benny Sudakov. Dependent random choice. *Random Structures & Algorithms*, 38(1-2):68–99, 2011. URL: `https://arxiv.org/abs/0909.3271`.

[4] Justin Gilmer. A constant lower bound for the union-closed sets conjecture. *arXiv preprint arXiv:2211.09055*, 2022. URL: `https://arxiv.org/abs/2211.09055`.

[5] Stasys Jukna. *Extremal combinatorics: with applications in computer science*, volume 571. Springer, 2011.

[6] Gil Kalai. Blog post: union-closed sets conjecture, 2022. URL: `https://gilkalai.wordpress.com/2022/11/17/amazing-justin-gilmer-gave-a-constant-lower-bound-for-the-union-closed-sets-conjecture/`.

[7] Imre Leader. Ramsey theory lecture notes, 2000. URL: `https://www.dpmms.cam.ac.uk/~par31/notes/ramsey.pdf`.

[8] Yufei Zhao. *Graph theory and additive combinatorics: exploring structure and randomness*. Cambridge University Press, 2023. URL: `https://yufeizhao.com/gtacbook/gtacbook.pdf`.