MATH 4032, SPRING 2024: PROJECT 2

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For this project, you will choose a topic related to probabilistic combinatorics and write an expository report on it. The purpose of the report should be akin to introductory lecture notes, where the target audience is another student in our class: provide motivation for studying the topics, any necessary background or definitions, guiding examples, and a main theorem or theorems and proofs.

1. Topics

1.1. **List of Potential Topics.** Here are some suggested topics. Some of these topics are quite specific (e.g. a proof of a main theorem), whereas some of them are quite broad (e.g. a general proof method).

(1) Alterations / Deletions Method (applications include Ramsey Theory, independent sets, combinatorial geometry, and sphere packings)
(2) Variance and The Second Moment Method (applications include prime factors, clique number and subgraph counts in random graphs, and distinct sum subsets)
(3) The Lovász Local Lemma (applications include Ramsey theory, sphere coverings)
(4) Stirling's Formula
(5) Polya's Theorem for Random Walks on $\mathbb{Z}^d$ via Stirling's Formula
(6) More on Random Graphs
    - Janson's Inequality and Clique Number
    - Expected Subgraph Counts (see Second Moment Method readings)
(7) Entropy and Counting (applications include intersecting set families, matchings, independent sets, and colorings)
(8) Probabilistic Games: Liar Game and Tenure Game
(9) Percolation Phase Transition on $\mathbb{Z}^2$

Along with each topic, I will provide some sample readings on Canvas to start your exploration. You may choose to stick primarily to the suggested reading or to look for other sources. The readings may contain much more information than you will be able to use in a report, so there are many different ways you can utilize them. For example, suppose you pick the Second Moment Method. You can give a broad overview of the method, or you can pick a specific example to focus on. Or, you can pick one example as the main report topic (e.g. Ramsey Theory) and compare the second moment results with other probabilistic techniques.

1.2. **General References.** The readings will be excerpted from the following:

- *The Probabilistic Method* by Alon and Spencer,
- *Extremal Combinatorics* by Jukna,
- *Random Graphs* by Frieze and Karonski,
- *Asymptopia* by Spencer and Florescu,
- Entropy Lecture Notes by Galvin,
- Percolation Lecture Notes by Sousi

All except the first have the full text available online. If you would like to reference a full copy of Alon-Spencer, you may scan mine during office hours.

1.3. **A Note on Topic Choice.** The topics above are suggested because I think they are the most accessible based on what we have covered in class and also lend themselves to a lot of specific examples. You are welcome to propose something else related to probabilistic combinatorics, but please let me know so we can discuss if it would be suitable for this project.

2. Requirements of Report

2.1. **Formatting.**

- Length: 2–4 pages
- Format: typed, single-spaced in 11–12pt font (this is the LaTeX default) with 1-inch margins
- I *strongly encourage you* to write this report in LaTeX but it is not required.
- At the end should be a bibliography. An easy way to do this is to create a separate .bib file (in the same folder on your computer, or in the same project on Overleaf) and then use the code (for example) `\bibliographystyle{plain} \bibliography{name of .bib file}`. Here are some instructions about how to do this.

2.2. **Content.**

- An introduction section, including motivation for studying the topic
- Any new definitions or notation that we have not introduced in class
- At least one "main theorem"
- At least one mathematical proof. Note: it doesn't have to be the proof of the main theorem if, for example, the main theorem proof is too complicated to discuss in full detail.
- A conclusion
- In general, you may research anything related to the topic but please cite all sources.
- Your target audience should be other students, such as those in Math 4032, so keep that in mind when deciding what background to include.

2.3. **Deadlines.**

- Due Friday, March 1 at 11:59pm: your choice of topic
- Due Friday, March 8 at 11:59pm: an outline of your report. Examples include a set of bullet points with the key facts or theorems you will describe, a list of references, placeholders for illustrations you will insert, etc.
- Due Friday, March 15 at 11:59pm: final project.

3. Grading

Here are the categories according to which your project will be graded. See Canvas for a full breakdown of the rubric.

3.1. **Mathematics.** (10 pts) This category is for grading the proofs and mathematical correctness of your report. Proofs and explanations will be graded roughly according to the "Proofs" rubric used for problem sets. Namely,

- Proofs and explanations are correct, flow logically, and are easy to follow without over- or under-explaining.
- Mathematical grammar and notation are used correctly.

3.2. **Exposition and Writing.** (10 pts) This category is for grading the "essay" aspects of your report.

- The report is cohesive and has a narrative structure, as opposed to simply a list of statements or theorems.
- The report is well-organized and appropriately uses sections or headings when needed.
- The writing is clear and engaging, and targets an appropriate audience.

3.3. **Content.** (10 pts) This category is for grading how well your report balances the above two categories.

- The content of the report is clearly related to the chosen topic.
- The topics discussed in the report are sufficiently related to the class content but are not rote repetition of class content. Topics outside of the scope of class are prefaced with an appropriate amount of introduction and motivation.
- There is an appropriate balance between exposition and proofs.

3.4. **Formatting and Grammar.** (10 pts)

- The report has correct spelling, punctuation, and English grammar.
- The report follows the formatting guidelines specified in this document.
- References are appropriately cited inline and are listed in a bibliography.

3.5. **Creativity (bonus).** (3 pts)

A straightforward report that satisfies all the criteria above will earn full credit. But bonus points will be awarded if you bring something "extra" to the table or go above and beyond in some way. It can take the form of anything—an illustration, an analogy, code, a poem. But it still has to be relevant to your topic: a song about the *arctan* function, while certainly creative, would not be relevant.