MATH 4032, SPRING 2024: PROJECT 1

For this project, you will pick an integer sequence from the OEIS (Online Encyclopedia of Integer Sequences) and write an expository report on it.

1. The OEIS

The OEIS `https://oeis.org/` is a database of integer sequences. It was started by Neil Sloane around 1964 as a physical database on notecards and transitioned to an online database in the 1990s. It currently contains over 367,000 sequences.

2. Requirements of Report

You should start by picking a sequence. You are free to pick one that is not suggested below, but if you do, please ask me about it first. There are thousands of options but not all of them will be suitable for this project. But you are welcome to pick things that are related to topics outside of combinatorics, such as graph theory, number theory, abstract algebra, etc.

You should then do some research (by following the links on the OEIS, by doing an internet search, or by looking in textbooks) on your sequence and write a short article about it. You will want to introduce the sequence, relate it to any concepts we've covered in class, and share some interesting mathematical facts about the sequence.

2.1. Suggested sequences to look at.

- the Catalan numbers* (`http://oeis.org/A000108`)
- the Narayana numbers/Catalan triangle (`https://oeis.org/A001263`)
- the Stirling numbers of the second kind* (`https://oeis.org/A008277`)
- the Stirling numbers of the first kind (`https://oeis.org/A008275`)
- the Lah numbers (`https://oeis.org/A008297`)
- the Bell numbers* (`https://oeis.org/A000110`)
- the Fibonacci numbers* (`http://oeis.org/A000045`)
- the tribonacci numbers (`https://oeis.org/A000073`)
- the triangular numbers (`https://oeis.org/A000217`)
- the Lucas numbers (`http://oeis.org/A000204`)
- the Stern-Brocot sequence (`http://oeis.org/A002487`)
- the Eulerian numbers (`https://oeis.org/A008292`)

* denotes sequences that we will touch on in class; if you pick one of these, your project should include additional things that we did not discuss in class (i.e. don't just summarize what we have already learned).

2.2. Formatting.

- Length: 2–4 pages
- Format: typed, single-spaced in 11–12pt font (this is the LaTeX default) with 1-inch margins
- I *strongly encourage you* to write this report in LaTeX but it is not required.
- At the end should be a bibliography. An easy way to do this is to create a separate .bib file (in the same folder on your computer, or in the same project on Overleaf) and then use the code (for example) `\bibliographystyle{plain} \bibliography{name of .bib file}`. Here are some instructions about how to do this.

2.3. **Content.**

- A definition of the sequence, and any accompanying description of the sequence. You can use examples, pictures, code, or anything that you would find useful to describe the sequence. You should include at least the first several terms of the sequence.
- Any connections of the sequence to concepts we've learned in this class
- At least one mathematical proof related to the sequence. It can be a proof of a recurrence or a proof of a closed form of the sequence using recurrence relation techniques or generating functions. It can relate the sequence to another sequence using a combinatorial or bijective proof. Or it can be something else! You do not need to come up with the proof on your own but you should write the proof in your own words.
- In general, you may research anything related to the sequence but please cite all sources.
- Your target audience should be other students, such as those in Math 4032, so keep that in mind when deciding what background to include.

2.4. **Deadlines.**

- Due Wednesday, January 24 at 11:59pm: your choice of sequence with a link to the OEIS entry; if you are not picking one of the suggested sequences and/or would like advice, feel free to email me.
- Due Friday, February 2 at 11:59pm: an outline of your report. Examples include a set of bullet points with the key facts or theorems you will describe, a list of references, placeholders for illustrations you will insert, etc.
- Due Friday, February 9 at 11:59pm: final project.

3. Grading

Here are the categories according to which your project will be graded.

3.1. **Mathematics.** (15 pts) This category is for grading the mathematical content of your report. Proofs and explanations will be graded roughly according to the "Proofs" rubric used for problem sets. Namely,

- Proofs and explanations are correct, flow logically, and are easy to follow without over- or under-explaining.
- Mathematical grammar and notation are used correctly.

3.2. **Exposition and Writing.** (15 pts) This category is for grading the "essay" aspects of your report.

- The report is cohesive and has a narrative structure, as opposed to simply a list of statements or theorems.
- The report is well-organized and appropriately uses sections or headings when needed.
- The writing is clear and engaging, and targets an appropriate audience.

3.3. **Formatting and Grammar.** (10 pts)

- The report has correct spelling, punctuation, and English grammar.
- The report follows the formatting guidelines specified in this document.
- References are appropriately cited inline and are listed in a bibliography.

3.4. **Creativity (bonus).** (3 pts)

A straightforward report that satisfies all the criteria above will earn full credit. But bonus points will be awarded if you bring something "extra" to the table or go above and beyond in some way. It can take the form of anything—an illustration, an analogy, code, a poem. But it still has to be relevant to your topic: a song about the *arctan* function, while certainly creative, would not be relevant.